Ньютон биномы тақырыбындағы күрделі есептерді шешу

$$(a+b)^n = C_n^0 a^n + C_n^1 a^{n-1} b + C_n^2 a^{n-2} b^2 + \ldots + C_n^n b^n$$

№1. $C_n^1 + C_n^2 + \ldots + C_n^{n-1} = a$ өрнегін ықшамдаңыз.

$$C_n^0 + C_n^1 + C_n^2 + \ldots + C_n^{n-1} + C_n^n = a + C_n^0 + C_n^n = a + 2$$

$$C_n^0 = \frac{n!}{n! \cdot 0!} = 1 = C_n^n$$

$$C_n^0 \cdot 1 + C_n^1 \cdot 1 + C_n^2 \cdot 1 + \ldots + C_n^{n-1} \cdot 1 + C_n^n \cdot 1 = \left| \begin{array}{l} a = 1 \\ b = 1 \end{array} \right| = (1 + 1)^n$$

$$a + 2 = 2^n \Rightarrow a = 2^n - 2$$

$$C_n^1 + C_n^2 + \ldots + C_n^{n-1} = 2^n - 2$$

$n! \cdot (n+1) = (n+1)!$

№2. $C_n^0 + \frac{1}{2}C_n^1 + \ldots + \frac{1}{n+1}C_n^n = \frac{2^{n+1}-1}{n+1}$ тепе-теңдігін дәлелдеңіз.

$$(n+1)\cdot\left(C_n^0 + \frac{1}{2}C_n^1 + \ldots + \frac{1}{n+1}C_n^n\right) = \boxed{2^{n+1}-1}$$

$$\frac{n!}{n!\cdot 0!}\cdot(n+1) + \frac{n!}{(n-1)!\,1!}\cdot\frac{n+1}{2} + \frac{n!}{(n-2)!\,2!}\cdot\frac{n+1}{3} + \ldots + \frac{n!}{1!(n-1)!}\cdot\frac{n+1}{n} + \frac{n!}{n!\cdot 0!}\cdot\frac{n+1}{n+1} = \boxed{\frac{(n+1)!}{n!\cdot 1!}} + \frac{(n+1)!}{(n-1)!\cdot 2!} + \frac{(n+1)!}{(n-2)!\cdot 3!} + \ldots$$

$$+\frac{(n+1)!}{1!\cdot n!} + \frac{(n+1)!}{0!\,(n+1)!} = \boxed{C_{n+1}^1 + C_{n+1}^2 + C_{n+1}^3 + \ldots + C_{n+1}^n} + C_{n+1}^{n+1}$$

$$2^{n+1} - 2 \qquad +1$$

$T_1, T_2, T_3, \ldots, T_{21}$ $\quad T_2 > T_1$ $\quad T_3 > T_2$

$x \in [1; 20]$

№3. $(\sqrt{5}+\sqrt{2})^{20}$ жақшаны толық ашқандағы ең үлкен қосылғышты табыңыз.

$C_{20}^{0}\sqrt{5}^{20} + C_{20}^{1}\sqrt{5}^{19}\cdot\sqrt{2} + C_{20}^{2}\cdot\sqrt{5}^{18}\cdot\sqrt{2}^{2} + \ldots + C_{20}^{20}\sqrt{2}^{20}$

$T_1 = C_{20}^{0}\sqrt{5}^{20}\cdot\sqrt{2}^{0}$; $T_2 = C_{20}^{1}\sqrt{5}^{19}\cdot\sqrt{2}^{1}$

$T_x = C_{20}^{x-1}\sqrt{5}^{21-x}\sqrt{2}^{x-1}$; $T_{x+1} = C_{20}^{x}\sqrt{5}^{20-x}\sqrt{2}^{x}$; $T_{x+1} > T_x \Rightarrow \boxed{\frac{T_{x+1}}{T_x} > 1}$

$$\frac{C_{20}^{x}\sqrt{5}^{20-x}\sqrt{2}^{x}}{C_{20}^{x-1}\sqrt{5}^{21-x}\sqrt{2}^{x-1}} = \frac{\frac{20!}{x!\,(20-x)!}\sqrt{2}}{\frac{20!}{(x-1)!\,(21-x)!}\sqrt{5}} = \frac{21-x}{x}\cdot\frac{\sqrt{2}}{\sqrt{5}} > 1 \Rightarrow (21-x)\sqrt{2} > x\sqrt{5}$$

$21\sqrt{2} - x\sqrt{2} > x\sqrt{5} \Rightarrow 21\sqrt{2} > x(\sqrt{5}+\sqrt{2}) \Rightarrow \frac{21\sqrt{2}}{\sqrt{5}+\sqrt{2}} > x \Rightarrow x < \frac{21\sqrt{2}(\sqrt{5}-\sqrt{2})}{5-2} \Rightarrow$

$x < 7(\sqrt{10}-2) \approx 8,1$ $\quad \boxed{x \leq 8}$

$a < 1$

$T_{x+1} > T_x$ $\quad T_1 < T_2 < T_3 < T_4 < T_5 < \ldots < T_9 > T_{10}$

$T \quad 8 \quad 12 \quad 8$

$$\boxed{T_9 = C_{20}^{8}\sqrt[?]{5}^{?}\cdot\sqrt[?]{2}^{?}}$$

№4. $(2x+3y)^{30}$ ең үлкен коэффициентін табыңыз

$$T_n = C_{30}^{n-1}(3y)^{31-n}\cdot(2x)^{n-1};\quad T_{n+1} = C_{30}^{n}(3y)^{30-n}(2x)^{n}$$

$$T_{n+1} > T_n \Rightarrow \frac{T_{n+1}}{T_n} > 1 \Rightarrow \frac{C_{30}^{n}(3y)^{30-n}(2x)^{n}}{C_{30}^{n-1}(3y)^{31-n}(2x)^{n-1}} = \frac{\frac{\cancel{30!}}{\cancel{(30-n)!}\cdot\cancel{n!}_{n}}\cdot 2x}{\frac{\cancel{30!}}{\cancel{(31-n)!}_{31-n}\cdot\cancel{(n-1)!}}\cdot 3y} =$$

$$= \frac{(31-n)\cdot 2x}{n\cdot 3y}\qquad \frac{31-n}{n}\cdot\frac{2}{3} > 1 \Rightarrow (31-n)\cdot 2 > 3n$$

$$62-2n > 3n \Rightarrow 5n < 62$$

$$n < 12{,}4$$

$$n \le 12$$

$$T_n < T_{n+1}\quad n \le 12$$

$$T_{12} < T_{13}$$

Полиномдық формула

$$\boxed{(a+b+c)^3 = (a+b+c)\cdot(a+b+c)(a+b+c)}$$

$$(a+b)^4 = (a+b)(a+b)(a+b)(a+b)$$

a a a a

$$C_4^4\, a^4 \quad C_4^3 \boxed{C_1^1}\, a^3 b$$

$$(a+b+c)^3 = (C_3^3 a^3 + C_3^3 b^3 + C_3^3 c^3) + (C_3^2 a^2 b + C_3^2 b^2 c + C_3^2 c^2 a) +$$

$$\ldots + \boxed{C_3^1 C_2^1 \cdot C_1^1}\, abc$$

a a a

$$a, a, a - 1 \quad \ldots)^{3!}$$

$$(a+b+c)^3 = \begin{matrix} b & b & b \\ c & c & c \end{matrix}$$

$$\begin{matrix} a & b & a \\ a & a & b \\ b & a & a \end{matrix} \quad a^2 b \quad P(2,1) = \frac{\;}{2!\cdot 1!}$$

$$P(3) = \frac{3!}{3!}$$

$$P(3)\left(a^3 + b^3 + c^3\right)$$

$$P(2,1)\left(a^2 b + b^2 c + \ldots\right)$$

$$P(1,1,1)\, abc$$

$$\frac{3!}{1!\cdot 1!\cdot 1!}$$

$$(a+b+c+d)^5 = P(5)\left(a^5 + b^5 + c^5 + d^5\right)$$

$$P(4,1)\left(a^4 b^1 + b^4 a + c^4 a + \ldots\right)$$

$$P(3,1,1)\left(a^3 bc + \ldots\right.$$

$$P(2,2,1)\, a^2 b^2 c$$

$$(a+b)^n = P(n)\, a^n + P(n-1,1)\, a^{n-1} b + P(n-2,2)\, a^{n-2} b^2 + P(n-3,3)\, a^{n-3} b^3 + \ldots$$

$$+ P(n)\, b^n$$